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Analytic Solution for the problem of Optimization of the Bi-elliptic Transfer Orbits with Plane Change

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***Abstract*— This study gives the optimal total characteristic velocity required to transfer a space-vehicle between non-coplanar elliptic orbits having common center of attraction and collinear major axes (coaxial ellipses). The bi-elliptic transfer considered here consists of two semi-elliptic transfer orbits connecting the initial and final orbits which are considered ellipse also. For the optimal 3-impulse transfer, we have 3-plane changes, with angles α, β, γ respectively with their sum as the total inclination of the final to the initial orbital planes. The method were illustrated though two examples.**

***Index Terms*— Space maneuvers, Bielliptic Transfer, Optimization , Plane Change.**

I. INTRODUCTION

The determination of a specific orbit and the procedure to calculate orbital maneuvers for artificial satellites are problems of extreme importance in the study of orbital mechanics. Therefore, the transferring problem of a spaceship from one orbit to another and the attention to this subject has increased during the last years. R. H.Goddard [1] was one of the first researchers to work on the problem of optimal transfers of a spacecraft between two points. He proposed optimal approximate solutions for the problem of sending a rocket to high altitudes with minimum fuel consumption. After him comes the very important work done by Hohmann [2]. He solved the problem of minimum ΔV transfers between two circular coplanar orbits. His results are largely used nowadays as a first approximation of more complex models and it was considered the final solution of this problem until 1959. A detailed study of this transfer can be found in Marec [3] and an analytical proof of its optimality can be found in Barrar [4]. The Hohmann transfer would be generalized to the elliptic case (transfer between two coaxial elliptic orbits) by Marchal [5]. Smith [6] shows results for some other special cases, like coaxial and quasicoaxial elliptic orbits, circular-elliptic orbits, two quasi-circular orbits. Hohmann type transfers between non-coplanar orbits are discussed in several papers, like McCue [7], that study a transfer between two elliptic inclined orbits including the possibility of rendezvous; or Eckel and Vinh [8] that solve the same problem with time or fuel fixed. More recently, the literature studied the problem of a two-impulse transfer where the magnitude of the two impulses are fixed, like in Jin and Melton [9]; Jezewski and Mittleman [10]. Later, the three-impulse concept was introduced in the literature. Using this concept, it is possible to show that a bi-elliptical transfer between two circular orbits has a lower ΔV than the Hohmann transfer, for some combinations of initial and final orbits. Roth [11] obtained the minimum ΔV solution for a bi-elliptical transfer between two inclined orbits. Following the idea of more than two impulses, we have the work done by Prussing [12] that admits two or three impulses; Prussing [13] that admits four impulses; Eckel [14] that admits N – impulses.

In this study we shall obtain the optimal total characteristic velocity required to transfer a space-vehicle between non-coplanar elliptic orbits having common center of attraction and collinear major axes (coaxial ellipses). The bi-elliptic transfer considered here consists of two semi-elliptic transfer orbits connecting the initial and final orbits which are considered ellipse also. Therefore three impulses are required, and these are assumed to occur only at the apsides of the ellipses. For the optimal 3-impulses, we have 3-plane changes, with angles α, β, γ respectively with their sum as the total inclination of the final to the initial orbital planes. The first and the third are partial angles of plane change and are always small. Hence the second plane change β , is dominant, which takes place at the coincident apo-centers of the first and the second transfer orbits.

II. DESCRIPTION OF THE MANEUVER

It is required to transfer a space-vehicle which is moving in an elliptic orbit, with eccentricity e_1 , semi-major axis a_1 and peri-centre distance r_{1A} , to another non-coplanar elliptic orbit with parameters (e_2, a_2) and peri-centre distance r_{2C} ,

The two orbits having a common center of attraction and collinear major axes as shown in Fig.1. We have 4-feasible configurations of this maneuver, discussed in the planar case by Kamel, Ammar [15].

Here we shall consider the first configuration in which the transfer orbits O_T and $O_{T'}$ connect the peri-centre of the initial orbit O_1 and the peri-centre of the final orbit O_2 . The geometry of the maneuver is described in Fig.1.

From Fig. (1), we can obtain the following relations, resulting from the cotangential conditions at the apsides.

- At the peri-center A: $r_{1A} = r_{TA}$;

$$a_1(1-e_1) = a_T(1-e_T) \quad (1\text{- a})$$

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- At the apo-center B: $r_{TB} = r_{T'B}$;

$$a_T(1+e_T) = a_{T'}(1+e_{T'}) \tag{1- b}$$

- At the peri-center C: $r_{T'C} = r_{2C}$;

$$a_{T'}(1-e_{T'}) = a_2(1-e_2) \tag{1- c}$$

Also, we put

$$r_{2E} = a_2(1+e_2),\, r_D = a_1(1+e_1) \tag{1- d}$$

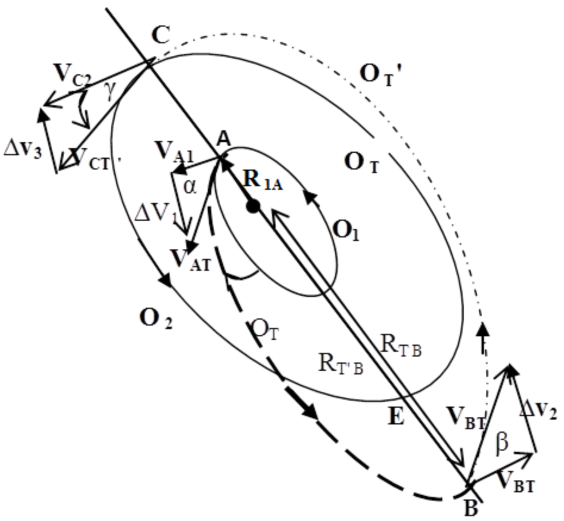

Fig. 1: Geometry of the maneuver

Referring to Fig. 1, and using the vis - visa equation

$$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

We can obtain the velocities at the apsides of the orbits as:

$$V_{1A}{}^2 = \mu\left(\frac{2}{r_A} - \frac{1}{a_1}\right) = \frac{2\mu}{r_A}\left(\frac{r_D}{r_A + r_D}\right) \tag{2-a}$$

$$V_{TA}{}^2 = \mu\left(\frac{2}{r_{TA}} - \frac{1}{a_T}\right) = \frac{2\mu}{r_A}\left(\frac{r_{TB}}{r_A + r_{TB}}\right) \tag{2-b}$$

$$V_{TB}{}^2 = \mu\left(\frac{2}{r_{TB}} - \frac{1}{a_T}\right) = \frac{2\mu}{r_A}\left(\frac{r_A}{r_{TB}}\right)\left(\frac{r_A}{r_A + r_{TB}}\right) \tag{2-c}$$

$$\begin{aligned} V^2_{T'B} &= \mu\left(\frac{2}{r_{T'B}} - \frac{1}{a_{T'}}\right) \\ &= \frac{2\mu}{r_A}\left(\frac{r_A}{r_{TB}}\right)\left(\frac{r_{2C}}{r_{2C} + r_{TB}}\right) \end{aligned} \tag{2- d}$$

$$\begin{aligned} V_{T'C}{}^2 &= \mu\left(\frac{2}{r_{T'C}} - \frac{1}{a_{T'}}\right) \\ &= \frac{2\mu}{r_A}\left(\frac{r_A}{r_{2C}}\right)\left(\frac{r_{TB}}{r_{2C} + r_{TB}}\right) \end{aligned} \tag{2- e}$$

$$\begin{aligned} V_{2C}{}^2 &= \mu\left(\frac{2}{r_{2C}} - \frac{1}{a_2}\right) \\ &= \frac{2\mu}{r_A}\left(\frac{r_A}{r_{2C}}\right)\left(\frac{r_{2E}}{r_{2C} + r_{2E}}\right) \end{aligned} \tag{2- f}$$

Where $\mu = GM$, is the gravitational constant of the celestial body in our problem?

Now, we calculate the three required impulses for the maneuver:

(i) The first impulse (ΔV_1) is applied at the peri-center A of the initial orbit at an angle α to the initial plane, which changes the orbital velocity from V_{1A} to V_{TA}, causing a rotation of the orbital plane of the first transfer orbit O_T by the angle α about the line of nodes (co-axial major axis of the orbits). This impulse is given by

$$\begin{aligned} (\Delta V_1)^2 &= V_{1A}^2 + V_{TA}^2 - 2V_{1A}V_{TA}\cos\alpha \\ &= \frac{2\mu}{r_A}\cdot\left[\frac{r_D}{r_A + r_D} + \frac{r_{TB}}{r_A + r_{TB}} \right. \\ &\quad \left. -2\sqrt{\left(\frac{r_D}{r_A + r_D}\right)\left(\frac{r_{TB}}{r_A + r_{TB}}\right)}\cos\alpha\right] \end{aligned} \tag{3}$$

(ii) The second impulse (ΔV_2) is applied at the apo-center B of the first transfer orbit at an angle β to the plane, which changes the orbital velocity from V_{TB} to $V_{T'B}$, causing a rotation of the orbital plane of the second transfer orbit by an angle β about the line of nodes. This impulse is given by

$$\begin{aligned} (\Delta V_2)^2 &= V_{TB}^2 + V_{T'B}^2 - 2V_{TB}V_{T'B}\cos\beta \\ &= \frac{2\mu}{r_A}\cdot\frac{r_A}{r_{TB}}\left[\frac{r_A}{r_A + r_{TB}} + \frac{r_{2C}}{r_{2C} + r_{TB}} \right. \\ &\quad \left. -2\sqrt{\left(\frac{r_A}{r_A + r_{TB}}\right)\left(\frac{r_{2C}}{r_{2C} + r_{TB}}\right)}\cos\beta\right] \end{aligned} \tag{4}$$

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(iii) Finally, The third impulse (ΔV_3) is applied at the peri-center C of the second transfer orbit $O_{T'}$ making an angle γ with the orbital plane, which changes the orbital velocity from $V_{T'C}$ to V_{2C}, causing a rotation of the final orbit O_2 by an angle γ about the nodal line and transferring the vehicle to the target orbit. This impulse is given by

$$\begin{aligned}(\Delta V_3)^2 &= V_{T'C}^2 + V_{2C}^2 - 2V_{T'C}V_{2C}\cos\gamma \\ &= \frac{2\mu}{r_A}\cdot\frac{r_A}{r_{2C}}\left[\frac{r_{TB}}{r_{2C}+r_{TB}} + \frac{r_{2E}}{r_{2C}+r_{2E}}\right. \\ &\left. -2\sqrt{\left(\frac{r_{TB}}{r_{2C}+r_{TB}}\right)\left(\frac{r_{2E}}{r_{2C}+r_{2E}}\right)}\cos\gamma\right]\end{aligned} \qquad (5)$$

Such that we use relations (1) and (2) to get the equations (3), (4) and (5).

III. ANALYTIC SOLUTION

To simplify the notations, we shall introduce new variables x, y defined as:

$$x = \frac{r_{TB}}{r_A} = \frac{1+e_T}{1-e_T} \quad , \quad y = \frac{r_{T'B}}{r_{2C}} = \frac{1+e_{T'}}{1-e_{T'}} \qquad (6)$$

Also, introducing the new constants k, h and g, defined as:

$$k = r_A / r_{2C} \quad , \quad g = 1+e_2 \quad , \quad h = 1+e_1 . \qquad (7)$$

The three impulses in terms of the new notations can be written as:

$$\Delta V_1^2 = V_C^2\left[h + \frac{2x}{1+x} - 2\sqrt{\frac{2hx}{1+x}}\cos\alpha\right] \qquad (8)$$

$$\Delta V_2^2 = V_C^2\left(\frac{2}{x}\right)\left[\frac{1}{1+x} + \frac{1}{1+y} - 2\sqrt{\frac{1}{(1+x)(1+y)}}\cos\beta\right] \qquad (9)$$

$$\Delta V_3^2 = 2V_C^2\left(\frac{y}{x}\right)\left[\frac{y}{1+y} + \frac{1}{1+g} - 2\sqrt{\frac{1}{1+g}}\sqrt{\frac{y}{1+y}}\cos\gamma\right] \qquad (10)$$

Where V_C is the circular velocity at peri- center A, of the initial elliptic orbit, given by

$$V_C^2 = \frac{\mu}{r_A} = \frac{\mu}{a_1(1-e_1)} \qquad (11)$$

The two variables x , y are not independent, and we can obtain the relation between them as :

$$y = \frac{r_{TB}}{r_{2C}} = \frac{r_{TB}}{r_A}\times\frac{r_A}{r_{2C}} = k\,x \qquad (12)$$

The total characteristic velocity

$$\Delta V_T = \left(\sum_{i=1}^{3}\Delta V_i\right)/V_C$$

could be written in terms of x and the angles α, β, γ as :

$$\begin{aligned}\Delta V_T &= \sqrt{h + \frac{2x}{1+x} - 2\sqrt{\frac{2hx}{1+x}}\cos\alpha} \\ &+ \sqrt{\frac{2}{x}}\sqrt{\frac{1}{1+x} + \frac{1}{1+kx} - 2\sqrt{\frac{1}{(1+x)(1+kx)}}\cos\beta} \\ &+ \sqrt{k}\sqrt{g + \frac{2kx}{1+kx} - 2\sqrt{\frac{2gkx}{1+kx}}\cos\gamma}\end{aligned} \qquad (13)$$

Thus, we can write

$$\Delta V_T = \sqrt{A(x,\alpha)} + \sqrt{B(x,\beta)} + \sqrt{k\,C(x,\gamma)} \qquad (14)$$

where

$$A(x,\alpha) = h + \frac{2x}{1+x} - 2\sqrt{\frac{2hx}{1+x}}\cos\alpha \qquad (15)$$

$$B(x,\beta) = \left(\frac{2}{x}\right)\left[\frac{1}{1+x} + \frac{1}{1+kx} - 2\sqrt{\frac{1}{(1+x)(1+kx)}}\cos\beta\right] \qquad (16)$$

$$C(x,\gamma) = g + \frac{2kx}{1+kx} - 2\sqrt{\frac{2gkx}{1+kx}}\cos\gamma \qquad (17)$$

We shall use the Lagrange's multiplier technique to obtain the extreme values for the total impulse required for the maneuver. Thus define the characteristic function $F(x,\alpha,\beta,\gamma)$ as:

$$F(x,\alpha,\beta,\gamma) = V_T(x,\alpha,\beta,\gamma) + \lambda\cdot(\theta - \alpha - \beta - \gamma) \qquad (18)$$

where θ, is the total inclination between the initial and final orbits, and λ is the Lagrange's multiplier. The optimization

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conditions that minimize the total impulse are:

$$\frac{\partial F}{\partial x}=0\ ,\quad \frac{\partial F}{\partial \alpha}=0\ ,\quad \frac{\partial F}{\partial \beta}=0 \text{ and } \frac{\partial F}{\partial \gamma}=0 \qquad (19)$$

Using (14), we obtain the optimization condition of the characteristic function with respect to x as:

$$\frac{\partial F}{\partial x}=\frac{\partial A/\partial x}{2\sqrt{A}}+\frac{\partial B/\partial x}{2\sqrt{B}}+\sqrt{k}\,\frac{\partial C/\partial x}{2\sqrt{C}}=0 \qquad (20)$$

Using (15)-(17), we obtain:

$$\begin{aligned}\frac{\partial F}{\partial x}&=\frac{1}{\sqrt{A}}\left[\frac{\sqrt{2x}-\sqrt{h}\sqrt{1+x}\cos\alpha}{\sqrt{2x}\,(1+x)^2}\right]\\&+\frac{\sqrt{k}}{\sqrt{C}}\left[\frac{2k\sqrt{x}-\sqrt{g\,k}\sqrt{1+kx}\cos\gamma}{\sqrt{x}\,(1+kx)^2}\right]\\&-\frac{1}{\sqrt{B}}\left[\frac{(1+kx)^2+k(1+x)^2}{x\,(1+x)^2(1+kx)^2}-\frac{(1+k+2kx)\sqrt{(1+x)(1+kx)}\cos\beta}{x\,(1+x)^2(1+kx)^2}\right]\\&-\frac{1}{\sqrt{B}}\left[\frac{(1+kx)+(1+x)}{x^2(1+x)(1+kx)}-\frac{2\sqrt{(1+x)(1+kx)}\cos\beta}{x^2(1+x)(1+kx)}\right]=0\end{aligned} \qquad (21)$$

The optimization condition of the characteristic function with respect to the inclinations i.e. the distribution of the plane change among the three maneuvers according to the optimality conditions, are given by:

$$\frac{\partial F}{\partial \alpha}=\frac{\sqrt{\frac{2hx}{1+x}}\sin\alpha}{\sqrt{A}}-\lambda=0 \qquad (22)$$

$$\frac{\partial F}{\partial \beta}=\frac{\frac{2}{x}\sqrt{\frac{1}{(1+x)(1+k\,x)}}\sin\beta}{\sqrt{B}}-\lambda=0 \qquad (23)$$

$$\frac{\partial F}{\partial \gamma}=\frac{\sqrt{\frac{2g\,k^2x}{1+k\,x}}\sin\gamma}{\sqrt{C}}-\lambda=0 \qquad (24)$$

Where we have used the relation: $\alpha+\beta+\gamma=\theta$. From the above equations we can obtain the relations:

$$\begin{aligned}&\frac{1}{\sqrt{A}}\sqrt{\frac{2hx}{1+x}}\sin\alpha\\&=\frac{1}{\sqrt{B}}\left(\frac{2}{x\sqrt{(1+x)(1+k\,x)}}\right)\sin\beta\\&=\frac{k}{\sqrt{C}}\sqrt{\frac{2g\,x}{1+k\,x}}\sin\gamma\end{aligned} \qquad (25)$$

Since we have assumed that α and γ are small, then without a loss of generality we can take as a simplification the case where $\alpha=\gamma$. In this case we have

$$\beta=\theta-2\alpha \qquad (26)$$

Equating (22), and (24), we obtain after simplifications:

$$k\sqrt{g}\sqrt{1+x}\sqrt{A}=\sqrt{h}\sqrt{1+k\,x}\sqrt{C} \qquad (27)$$

Substituting the values of A and C, squaring, and solving for cos α in terms of x, we obtain:

$$\cos\alpha=\frac{hg(1-k^2)+[hk(g+2)-k^2g(2+h)]x}{2h\sqrt{2gkx}\sqrt{1+kx}-2gk^2\sqrt{2hx}\sqrt{1+x}} \qquad (28)$$

To obtain cos β, in terms of x, we put

$$\cos\alpha=\xi(x)\,,\ \cos\beta=\eta(x)\,,$$
$$p=\cos\theta\,,\ q=\sin\theta$$

Knowing that

$$\cos\beta=\cos(\theta-2\alpha)=p\cos 2\alpha+q\sin 2\alpha$$

Then

$$\begin{aligned}\cos\beta=\eta(x)&=p\cdot[2\xi^2(x)-1]\\&+2q\cdot\xi(x)\sqrt{1-\xi^2(x)}\end{aligned} \qquad (29)$$

Substituting cos α , cos β from (28) , (29) into (13) and (21), we obtain the total characteristic impulse V_T and dV_T/dx in terms of x as:

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$$\Delta V_T = \left(h + \frac{2x}{1+x} - 2\sqrt{\frac{2hx}{1+x}}\,\xi(x)\right)^{1/2} + \left(\frac{2}{x}\right)^{1/2}\left(\frac{1}{1+x} + \frac{1}{1+kx} - 2\sqrt{\frac{1}{(1+x)(1+kx)}}\,\eta(x)\right)^{1/2} + k^{1/2}\left(g + \frac{2kx}{1+kx} - 2\sqrt{\frac{2gkx}{1+kx}}\,\xi(x)\right)^{1/2}$$

(30)

$$\frac{\partial V_T}{\partial x} = \frac{1}{\sqrt{A}}\left[\frac{\sqrt{2x} - \sqrt{h}\,\sqrt{1+x}\,\xi(x)}{\sqrt{2x}\,(1+x)^2}\right] + \frac{\sqrt{k}}{\sqrt{C}}\left[\frac{2k\sqrt{x} - \sqrt{g\,k}\,\sqrt{1+kx}\,\xi(x)}{\sqrt{x}\,(1+kx)^2}\right]$$

$$-\frac{1}{\sqrt{B}}\left[\frac{(1+kx)^2 + k(1+x)^2}{x\,(1+x)^2(1+kx)^2} - \frac{(1+k+2kx)\sqrt{(1+x)(1+kx)}\,\eta(x)}{x\,(1+x)^2(1+kx)^2}\right]$$

$$-\frac{1}{\sqrt{B}}\left[\frac{(1+kx)+(1+x)}{x^2(1+x)(1+kx)} - \frac{2\sqrt{(1+x)(1+kx)}\,\eta(x)}{x^2(1+x)(1+kx)}\right] = 0$$

(31)

IV. NUMERICAL EXAMPLES

Example 1: (Earth – Pluto):
We shall consider the bielliptic transfer from Earth to Pluto. We shall use the following data:

a_1= 1 A.U, a_2 = 39.35 A.U, e_1 = 0.01671022 ,
e_2 = 0.24880766, $\theta = 17.1417^0$.

The value of x, which causes the minimum total characteristic velocity ΔV_T, can be calculated by solving equation (31). The graph of the total characteristic velocity is shown in Fig. 2

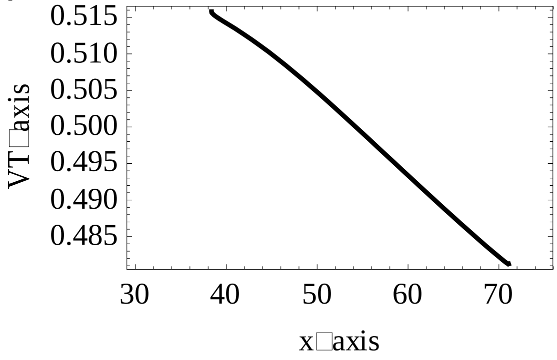

Fig .2: Total impulse VT vs x for Earth-Pluto bielliptic transfer

The graph of the derivative of total characteristic velocity with respect to x, is shown in Fig. 3.

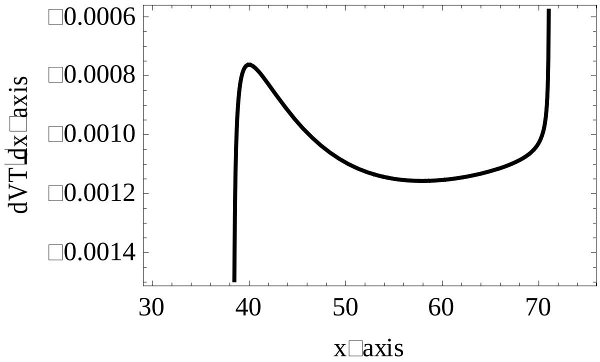

Fig .3: Derivative of total impulse VT vs x for Earth-Pluto bielliptic transfer

Solving equation (31) numerically and substitution into (28) - (30), we obtain the solution,

x = 71.125, β = 16.80, α = γ = 0.1670.

Solving Equations (1) and using (6), we can obtain the elements of the transfer orbits action as a function of x:

$$e_T = \frac{x-1}{x+1} = 0.97227\,, \qquad e_{T'} = \frac{k\,x-1}{k\,x+1} = 0.4058\,,$$

$$a_T = \frac{r_{1A}}{2}(x+1) = 34.4766\,A.U\,.$$

$$a_{T'} = \frac{r_{2C}}{2}(k\,x+1) = 49.7479\,A.U\,.$$

The total characteristic velocity is found to be equals to $\Delta V_T = 0.481211$

Example 2: (Circular – Geocentric transfer) We shall consider the bi-elliptic transfer from a geocentric circular orbit of radius 7000 km to one of radius 140 000 km , and the inclination between the two orbits is $\theta = 28.5^o$. The graph of the total characteristic velocity with respect to x, is shown in Fig.4, and the graph of the derivative of total characteristic velocity with respect to x, is shown in Fig.5. Solving equation (31) numerically, we obtain the solution, x = 26.3429. Substitution into (28) and (29) , we obtain the required angles for plane changes as:

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$\alpha = \gamma = 0.4^{o}$, $\beta = 27.65^{o}$.

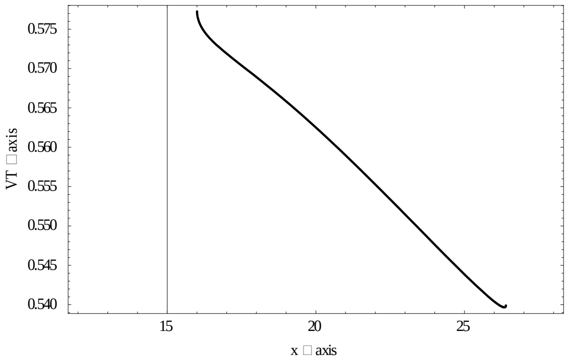

Fig. 4: Total impulse V_T vs x for circular – geocentric bielliptic transfer

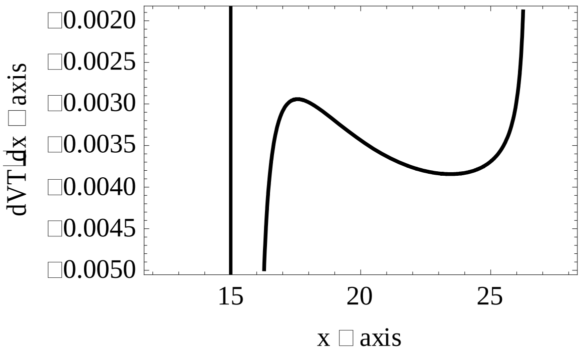

Fig .5:Total impulse dVT/dx vs x for circular-geocentric bielliptic transfer

Hence, we can obtain the elements of the transfer ellipses as:

$e_T = 0.92685$, $e_{T'} = 0.13687$,

$a_T = 95700.15km$, $a_{T'} = 162200.15km$,

The total characteristic velocity is found to be equal to

$\Delta V_T = 0.535664$.

V. CONCLUSION

An analytical formulation was derived and implemented, to solve the problem of three impulsive orbital transfers between elliptical non-coplanar orbits in a Keplerian dynamics problem with minimum fuel consumption. A numerical algorithm was developed for fast practical use to obtain the minimum velocity increment needed to perform this type of maneuver. The paper introduces a simple and straight forward algorithm for calculating the elements of the transfer orbits and change plane angles required for optimization.

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